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FIRST GRADE CHILDREN'S CONCEPT OF ADDITION OF NATURAL NUMBERS.

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MIDDLE-CLASS, FIRST-GRADE STUDENTS (100) WERE TESTED INDIVIDUALLY ON 4 ITEMS OF CONCEPT OF ADDITION AND CONSERVATION OF NUMBER. THE TEST ITEMS WERE IDENTICAL EXCEPT FOR THE NUMBER OF OBJECTS INVOLVED. FOR EACH ITEM, TWO PILES OF CANDY WERE PLACED BEFORE EACH CHILD AND THEN MOVED TOGETHER. THE STUDY SHOWED NO MAJOR DIFFERENCE IN THE MEAN PERFORMANCES OF THE CHILDREN AMONG SCHOOLS, SEXES, AND ITEMS. THESE FINDINGS WERE COMPARED TO THE SUBJECTS' SCORES ON A CONVENTIONAL PAPER-AND-PENCIL ADDITION TEST. IT WAS CONCLUDED THAT THE SUBJECTS HAD NOT ABSTRACTED THE CONCEPT OF THE SUM OF TWO WHOLE NUMBERS FROM PHYSICAL SITUATIONS, BUT HAD SIMPLY MEMORIZED THE ADDITION COMBINATIONS (FACTS). FURTHER STUDIES WERE SUGGESTED TO DETERMINE IF THESE CONCLUSIONS WOULD BE OPERATIVE ACROSS THE CONTINUUM OF CULTURAL LEVELS. (GD)

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Technical Report No. 5

FIRST GRADE CHILDREN'S CONCEPT OF ADDITION OF NATURAL NUMBERS

Henry Van Engen and Leslie P. Steffe

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for Learning and Re-Education
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Madison, Wisconsin
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PREFACE

The research and development program of the R & D Center is focused on improving efficiency of learning by children, youth, and adults. Efficiency of learning in school settings is related to six main groups of variables: the characteristics and behaviors of the learners, conditions of learning, characteristics and behaviors of the teacher, the subject matter, instructional materials and media, and forces outside the classroom. In this technical report relationships among variables pertaining to the learners and subject matter are reported. Specifically relationships of sex and IQ level to the acquisition of the concepts of addition and conservation of numerousness were ascertained.

This is the first technical report of the R & D Center that deals exclusively with concepts in a subject matter field. Other studies are underway in mathematics and also in other subject fields. A total instructional program in first grade mathematics comprised of video tapes, pupil exercises, and teacher notes is being developed and field-tested on a small scale during 1965-1966. This program will be given extensive field testing during 1966-1967 and will very likely be available for distribution nationally in 1967-1968. These research and development activities directed by Professor Henry Van Engen exemplify the R & D plan of simultaneously extending knowledge through research and improving the quality of learning through the development of instructional materials and procedures.

Herbert J. Klausmeier
Co-Director for Research

March 1, 1966

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ABSTRACT

One hundred first grade pupils from five schools serving a middle-class population were individually tested on four concept-of-addition items. For each item, two piles of candy were placed before the child and then moved together. The child was asked if he had a preference for either the two separate piles or the combined pile and why he had answered as he did. After responding to all four items, the child took a paper and pencil test that included sums representing two of the test items.

Analysis of variance showed no significant differences between boys and girls, among pupils from the five schools, or among items which varied in number of candies used (complexity).

The request for the child to state his preference, if he had one, for the separate or combined piles of candies had been made in two different ways. A χ^2 test showed that the two questions did not have significantly different effects on the child's responses.

A significant relationship was found between IQ and ability to abstract the concept of addition of whole numbers from the physical situation. Children scoring higher than chance on the four tasks had previously obtained higher IQ's than children scoring at or below chance level.

Scores on a paper and pencil test, which was used as a means of determining the relationship between knowledge of arithmetic facts and understanding of the concept of addition, indicated that nearly all pupils knew the sums $2+3=5$ and $4+5=9$. However, only 54 correctly stated no preference for separate or combined piles of candies when five candies were used, and only 45 when nine candies were used.

Two possible interpretations of the results were offered, the more logically acceptable being that children at the first grade level have not abstracted the concept of addition from physical situations but have memorized the addition facts. Implications for arithmetic instruction were discussed, and a number of researchable questions were raised.

I

INTRODUCTION

The sum of two whole numbers can be defined in terms of the union of two sets. Many arithmetic programs for the elementary school use this definition as a model when introducing addition of whole numbers. Such programs direct the children to consider first two collections of objects and then a new collection formed by combining the two original collections into a single collection, where the number of objects in the single collection is the sum of two numbers associated with the two original collections. The combining may take place physically, using objects, or conceptually. If it takes place physically, then the concept of conservation of numerousness is immediately involved when forming the sum that represents the number of objects in the single collection.

Conservation of numerousness means that, irrespective of how a set of objects is rearranged, the number of objects remains the same. In The Child's Conception of Number, Jean Piaget (1952) set forth the following three stages for the child's acquisition of this concept:

1. Absence of conservation. A child is totally unable to ignore his perceptions. He may think that there are more objects in the single collection, perhaps because it is more dense. Or he may think that there are more objects in the two collections, perhaps because they are farther apart.
2. Intermediary reactions. This is a transition stage from Stage 1 to Stage 3. In some cases, the child may give what can be termed Stage 3 responses, and in others, the child may give what can be termed Stage 1 responses.
3. Necessary conservation. A child is able to ignore his perception. He knows that the number of objects in the single collection is the same as the number of objects in the two original collections because "none has been taken away or added" or perhaps because "they are the same objects."

Of these stages, Piaget (1964) stated, "The ordering . . . is constant and has been found in all the societies studied. . . . However, although the order of succession is constant, the chronological ages of these stages varies a great deal (p. 10)."

Dodwell (1960), using 250 children ranging in age from 5 years 6 months to 8 years 10 months, found a low but significant negative correlation between the IQ's of the first and second graders involved and the Stage 1 responses given in his five tasks: (1) relation of perceived size to number (conservation of numerousness), (2) provoked correspondence, (3) unprovoked correspondence, (4) seriation, and (5) ordination and cardination. In the case of conservation of number (numerousness), only 50% of the children at 6 years 5 months exhibited Stage 3 responses, 80% at 7 years 6 months exhibited Stage 3 responses, and 80% at 8 years 6 months exhibited Stage 3 responses. For this phase of his study, he used two beakers of different diameter and had the child put, one by one, the same number of beads in each beaker. He then asked the child to make numerical comparisons between the beads in the two beakers.

A study by Kenneth Feigenbaum (1963) suggests that the number of objects in the collection may affect the child's ability to ignore his perception. Feigenbaum's population consisted of 90 children whose ages ranged from four to seven years. The children were placed into three experimental groups that received the same treatments with different materials. The first treatment involved a correspondence test using 28 beads and two glasses of the same size, a conservation test using the same beads (the beads were

poured into a smaller glass), a test for the understanding of "more" and "bigger," and finally a further correspondence and conservation test. Treatment II was the same as Treatment I except 14 beads instead of 28 were used. Treatment III was the same as Treatment II except the size differential of the glasses used in the conservation test was smaller. Feigenbaum cited evidence that the complexity of the stimuli (28 vs. 14 beads) affected the subjects' frequency of success in cases of incomplete assimilation of the principle of 1-1 correspondence.

From Dodwell's study and Piaget's three developmental stages, it appears that many children at the first grade level are not able to ignore their perception when responding to questions about the combining of two groups of physical objects and that IQ has relevance for this ability. Moreover, as Feigenbaum suggested, the number of objects in the collection may influence the ability of the children to ignore their perception. As Piaget has stated, children in all societies studied go through the three stages in understanding conservation of numerousness but perhaps at different rates and at different ages. It is interesting to explore the possibility of differences in the ability to ignore perception among children in different schools within the same school system.

The nine hypotheses tested in this experiment are:

(a) There is no significant difference among the mean scores on the four test items.
(b) There is no significant difference among the mean scores of pupils from the five schools.
(c) There is no significant difference between the mean scores of boys and girls.
(d) There is no significant interaction between schools and sex.
(e) There is no significant interaction between schools and test items.
(f) There is no significant interaction between sex and test items.
(g) There is no significant interaction among schools, sex, and test items.
(h) There is no significant correlation between IQ and the total scores on the test items or between IQ and the individual items.
(i) There is no significant effect on the success or failure of a test item from the manner in which the questions are asked.
(j) There is no significant correlation between children's knowledge of addition facts and their ability to perform related test items correctly.

II

METHOD

SUBJECTS

Subjects were 100 first grade pupils, 50 boys and 50 girls, randomly selected from five elementary schools in a large highly-supervised district. An additional 50 pupils were selected at the same time as alternates for any possible absentee or rejection, as described below. The children were individually tested, by one experimenter, in their own schools. School officials had designated nine schools serving a middle-class population from which the five participating schools were randomly selected. IQ's were obtained from previously administered Kuhlmann-Anderson Intelligence Tests.

As a preliminary test to ascertain whether S was prejudiced against candy, two piles of candy were placed before S, one of twelve and one of seven. The E asked S: "If I let you take some candies home for your friends, would you take this pile (pointing to the pile of seven), or this pile (pointing to the pile of twelve), or would it make any difference to you which pile you took home? . . . Why?"

The crucial response to the above question is S's response to "Why?" If S gave a reason such as "It doesn't make any difference because my friends don't like candy," he was excluded from the experiment. But if he gave a reason such as (after taking the one of twelve) "Because it has more," he was accepted for the experiment. In any case, the exclusion or acceptance of any S was entirely based on E's judgment of the child's bias against candy. Regardless of the outcome above, E proceeded directly into the experiment.

MATERIALS AND PROCEDURES

In the diagram below, the solid circles represent two collections of hard butterscotch candy in cellophane wrappers and the dotted circles represent these two collections combined into one. The numerals in the circles represent the number of candies in that collection. The arrows denote that both groups were moved an equal distance.

Four test items were randomly administered on an individual basis to each child. The procedure for each test item was the same, so a general procedure will be recounted.

The E placed two piles of candies before S, stated, "Here are two piles of candies," and pointed to each one. After a seven second interval, E moved the two piles together and stated, "I put them together and made one pile." The E again waited seven seconds and then asked S one of the following two questions, Question a if the item was the first or third in the sequence of items and Question b if the item was second or fourth in the sequence of items.

Question a: If I let you take the candies for your friends, would you take the two piles of candy or the one pile of candy after I put them together, or would it make any difference? . . . Why?

Question b: If I let you take the candies for your friends, would you take the one pile of candy after I put them together or the two piles of candy, or would it make any difference? . . . Why?

For any item, if S's response was, "It doesn't make any difference," he received a score of 1. Any other response was scored 0. In the event S scored 0, his choice of the one pile or the two piles of candy was recorded. All responses to the "why" question were also recorded.

After the four test items were administered, a paper and pencil test involving the following nine addition facts was administered to each S.

(1) 2	(2) 2	(3) 5	(4) 1	(5) 4
+2	+3	+4	+8	+3
(6) 3	(7) 4	(8) 4	(9) 5	
+2	+5	+1	+3	

A response of "5" to Numbers 2 and 6 was considered evidence that S "knew" the correct response to the sum of 2 and 3. Likewise, a response of "9" to Numbers 3 and 7 was considered evidence that S "knew" the sum of 5 and 4. In the event a subject responded correctly to both 2 and 6, he was given a score of 1. A similar procedure was followed for 3 and 7. Numbers 1, 4, 5, 8, and 9 were not scored.

III

RESULTS

A three way analysis of variance (ANOVA) was used to test Hypotheses a to g. Main effects were (1) schools, (2) sex, and (3) test items (complexity), where each S received all test items. Correlation coefficients were used in testing Hypotheses h and j. A χ^2 test of significance was used to test Hypothesis i.

The results of the ANOVA are shown in Table 1. The main effects of schools, sex, and test items did not result in significantly different test scores. However, it is interesting to note the consistently superior scores of pupils in School 2 across the four items. In addition, since the three two-way interactions and the three-way interaction were not significant, there is no evidence for rejection of Hypotheses a-g.

Table 1

Analysis of Variance Table of Mean Squares, Degrees of Freedom, and F Ratios for the Factors of Item Complexity, Schools, and Sex

Source of variation		df	SS	MS	F
Between subjects					
Schools	(A)	4	4.06	1.02	1.64
Sex	(B)	1	.64	.64	1.04
A × B		4	2.21	.55	< 1
SUBJECT W/G		90	55.60	.62	
Within subjects					
Test Item	(C)	3	.81	.27	2.22
A × C		12	2.44	.20	1.67
B × C		3	.14	.05	< 1
A × B × C		12	.81	.07	< 1
C × SUBJ. W/G		270	32.80	.12	

Note: No significant differences at .05 level of significance.

For the purpose of correlating IQ with total socre, a dichotomized total score was used with 0, 1, or 2 in one category and 3 or 4 (higher than chance) in another. Average IQ's for each item and for the dichotomized total score are shown in Table 2. The point-biserial correlation of .22 between IQ and this dichotomized total score was significant at the .05 level, refuting the first part of Hypothesis h. On the basis of the correlations for the separate items with IQ, the last part of Hypothesis h can be rejected for Items 2 and 3 but not for Items 1 and 4. These findings seem to be, in general, consistent with those of Dodwell who found a low but significant negative correlation between IQ and Stage 1 responses. (A distribution of total scores by IQ is shown in Table 3.)

Table 2

Average IQ's and Correlation with Correct and Incorrect Responses on Each Item and with Dichotomized Total Score

Item	Average IQ: Correct response	Average IQ: Incorrect response	r_{pb}
1	107.9	106.6	.04
2	111.1	104.2	.22*
3	111.0	104.3	.21*
4	100.5	103.6	-.10
Dichotomized Total Score	111.8[a]	104.3[b]	.22*
Total Score	--	--	.24[c]

* $p < .05$

[a] Total score of 3 or 4.

[b] Total score of 0, 1, or 2.

[c] No significance test made.

Table 3

Distribution of Total Scores by IQ

IQ	Total score 0	1	2	3	4
145-126	1	2	1	2	3
125-121	2	4	3	2	3
120-116	3	2	1	1	4
115-111	1	0	0	1	6
110-106	5	8	0	0	3
105-101	2	1	4	1	3
100-96	2	5	0	1	3
95-91	3	3	0	0	1
90-86	3	1	0	0	1
85-65	4	2	1	1	1
Total	26	27	10	9	28

The manner in which the question was asked had no effect at all on the outcome of correct and incorrect responses as can be seen by the nearly identical frequencies of responses shown in Table 4 ($\chi^2_2 = .10$). Altogether, 72 of the 400 reasons for responses were in the "No Reason," "Don't Know" and "Other" categories, 45 of them being in the "No Reason" category, that is, no reason at all. Of the remaining 27 nonspecific responses, 8 were in the "Don't Know" category. In general then, about 12% of the responses, those in the "No Reason" category, could not be validated through the "why" question. These responses, and those in the "Don't Know" category, were not excluded from the experiment because no decision can be made relative to their validity. The entries in Table 5 indicate that 93% of the correct responses were given by children who seemed to know why they gave them and were able to verbalize the reason to the experimenter. The

Table 4

Analysis of Questions and Responses

Response	Question a	b
Incorrect and the One Pile	73	75
Incorrect and the Two Piles	34	32
Correct	94	92
Total	201	199

Table 5

Analysis of Reasons for Correct Responses

"Why" response	Percent giving response
Same Number	79.6
Same Candies	7.5
Just Know	5.9
No Reason	5.4
Other	1.6

children who gave 5.4% of the correct responses made no verbal response to the "why" question.

The frequencies of the two types of incorrect responses are given in Table 6 for each item. Analysis of the reasons given by children who made incorrect responses is shown in Table 7. It is interesting to note that none of the children gave the response of "Fewer." Of those children who chose the one pile,

Table 6

Frequencies of Incorrect Responses by Type of Error and Item

Error	Item 1	2	3	4	Total
One Pile	32	36	41	39	148
Two Piles	14	18	14	20	66

Table 7

Analysis of Reasons for Incorrect Responses

"Why" response	Per cent one pile	Per cent two piles
More	71.6	45.4
Larger	6.8	3.0
Fewer	0.0	0.0
Smaller	3.4	3.0
No Reason	12.2	25.8
Don't Know	2.7	6.1
Other	3.4	16.7

aside from the fact that in 12.2% of their responses they were not able to verbalize a reason, the results were well categorized. However, the responses of those who chose the two piles were not so well categorized. Some of the 16.7% of the responses in the "Other" category were not nonsense responses and could be interpreted although no interpretation was attempted. Some of these responses are listed below.

"Not so much together."
"Doesn't make any difference but I want the two piles."
"They are more spread out."
"They are separate."
"Like them."
"Use for a party."
"Divide easier."
"Make a lot together."

Considering the dichotomous responses (conservation of numerousness vs. no conservation of numerousness). it could be expected, since a child has one chance in two of making a correct response, that at least 50 correct responses would be present for each item. $\chi^2_3 = 2.6$ shows that the actual frequencies presented in Table 8 do not depart from the expected value of 50 by any more than a chance fluctuation. This is in harmony with Dodwell's finding that only 50% of the children studied at 6 years and 5 months of age exhibited Stage 3 responses. However, considering just those 72 children who had at least one item wrong, the frequency of correct responses, also in Table 8, falls considerably below the chance level ($\chi^2 = 36.2$, $p < .01$), indicating there was a factor involved other than chance which influenced the children's responses.

Knowledge of addition facts does not seem to be a good indicator of a child's ability to perform the related test items correctly. The theoretical frequency of total scores in Table 9 is based on chance responses to the test items, with the probability of a correct response being taken to be .5. A χ^2 of 168.7 with four degrees of freedom shows that the observed total scores depart grossly from the theoretical total scores. As nearly all of the pupils tested knew the basic addition facts of 2 + 3 = 5 and 4 + 5 = 9 (Table 10), the theoretical frequency of total scores in Table 9 might be considered unrealistic. If knowledge of addition facts affected performance, a total score of 0 or 1 would be almost nonexistent and the frequencies would be distributed among the total scores of 2, 3, and 4 with perhaps 2 receiving the most, then 3 and 4 in that order. On this basis for the expected frequencies of total scores, χ^2 would still be very large. In the event of taking an expected frequency of 20 for each total score, $\chi^2_4 = 10.5$, still significant at the .01 level of significance. Hypothesis j cannot be rejected on the basis of this χ^2 or on the basis of the essentially zero correlation between knowledge of addition facts and performance on related items.

Table 8

Frequency of Correct Responses to Separate Items

Group	Item 1	2	3	4
Total N = 100	54	45	45	42
Children having at least one wrong N = 72	26	17	17	14

Table 9

Theoretical and Actual Frequencies of Total Test Scores

Frequency	Total test score 0	1	2	3	4
Theoretical	6.25	25	37.5	25	6.25
Actual	26	27	10	9	28

Of those 27 children who received a total score of 1, 15 responded correctly to Item 1 as is shown in Table 11. However, of those 19 who made total scores of 2 or 3, the correct response frequency was fairly evenly distributed over the items. Table 12 shows that the frequency of correct responses on Trials 1 and 2 were a little higher than on Trials 3 and 4. This can be attributed to the fact that six of the children who had a total score of 2 responded correctly on the first and second trials. Otherwise, no observable differences in frequencies of correct responses on trials are present.

Means and standard deviations for both IQ's and total test scores are given in Table 13.

Table 10

Frequency of Correct Responses
For Paper and Pencil Test
Sums 2 + 3 and 4 + 5 By Sex

Group	2 + 3	4 + 5	Total
Males	49	49	98
Females	50	45	95
Total	99	94	193

Table 11

Frequency of Correct Responses On Items for Total Scores of 1, 2, and 3

	Item				
Total score	1	2	3	4	Total
1	15	3	7	2	27
2	5	6	3	6	20
3	6	8	7	6	27

Table 12

Frequency of Correct Responses On Trials for Total Scores of 1, 2, and 3

	Trial				
Total score	1st	2nd	3rd	4th	Total
1	8	9	4	6	27
2	9	7	1	3	20
3	8	4	8	7	27

Table 13

Means and Standard Deviations
Of IQ and Total Test Score

Measure	Mean	Standard deviation
IQ	107.3	15.61
Total Score	1.89	1.59

IV

DISCUSSION

Five schools were randomly selected from a group of nine schools that serve a middle class population. At the first grade level, 10 boys and 10 girls were randomly selected from each of the five schools. Four test items that involved the concept of addition and conservation of numerousness were individually administered to each child. The test items were identical except for the number of objects involved.

The study showed no difference in the mean performance of the children between schools, sex or test items. The ability, on the part of a child, to respond correctly to an addition combination, for example 2+3, seems to have little or no relation to his ability to ignore his perception when two groups of objects, one of 2 and one of 3, are physically transformed into a group of 5. Two possible interpretations may be given to this phenomenon: (1) the children did not understand what they were asked; or (2) the children have not abstracted the concept of the sum of two whole numbers from physical situations but have memorized the addition combinations.

With regard to the first interpretation, approximately 5/40 of the total responses to the "why" question were in the "no reason" or "don't know" categories, and approximately 2/40 of the responses were categorized in the "other" category. Although a large number of responses in the "other" category were interpretable, approximately 7/40 of the total responses were not validated by the "why" question. But, it must be noted that 14 of the 72 responses in the "no reason," "don't know," and "other" categories came from children who had the test item correct but couldn't state why. If a child had an item wrong and the response (or lack of it) to the "why" question fell into one of these three categories, whether he understood what he was asked is an open question. However, for the purposes of arithmetic instruction, the inability to understand what he was asked (indeed, if that is the case) is significant in itself. Of the remaining 184 correct responses, those children who gave them stated quite affirmatively that there were the "same number" or that they were "the same candies" or that they "just know." A similar statement could be made for those children who gave the remaining incorrect responses, thus indicating that they understood what they were asked.

From the above discussion, one can conclude that most of the children understood what they were asked (with the exception of a possible 70 of 400 responses). The other alternative that exists then is Statement 2: the children have not abstracted the concept of the sum of two whole numbers from physical situations but have memorized the addition combinations. This is of serious consequence when one considers that the children were tested after they had studied arithmetic for one year.

Since this study was conducted in a middle-class district, further studies would be necessary to determine whether the phenomena observed are operative across the continuum of cultural levels, and whether at these different cultural levels they are operative at the same age level. Moreover, questions can be raised relative to the problem-solving abilities of elementary school children who are working at various levels of abstraction with regard to the conservation of numerousness, the operation of addition, etc. There is also a question about a variety of experiences (and what experiences, if any) being sufficient to raise the level of abstraction of elementary school children with regard to the concepts of number and its operations and the generalization of these concepts. The latter two questions may also be considered in the context of cultural levels.

To answer some of the above questions, studies are now in progress relative to problem-solving abilities of first grade children at various stages in number comprehension and the type of experiences that may raise the level of abstraction of elementary school children with regard to the concepts of number.

APPENDIX

Raw Data Tables

School 1

Boys	Item 1	2	3	4	Paper and pencil test	IQ
1	0	0	0	0	2	117
2	1	0	0	0	2	121
3	0	1	0	1	2	144
4	0	0	0	0	2	116
5	1	0	0	0	2	96
6	1	1	1	1	2	109
7	0	0	0	0	2	84
8	0	0	1	0	2	133
9	1	0	0	0	2	70
10	1	0	0	0	2	87
Total	5	2	2	2		
Girls						
1	0	0	0	0	2	115
2	1	1	0	0	2	119
3	1	0	0	0	2	73
4	1	1	0	0	2	103
5	1	0	0	0	1	106
6	0	0	0	0	2	123
7	1	1	1	1	2	128
8	1	0	1	0	2	121
9	1	1	1	1	2	112
10	1	1	1	1	2	119
Total	8	5	4	3		

Note: For test items, 0 indicates incorrect response; 1 indicates correct response.

	Item				Paper and pencil test	IQ
	1	2	3	4		
			School 2			
Boys						
1	1	1	1	1	2	129
2	1	1	1	1	2	113
3	1	0	0	0	2	109
4	1	1	1	1	2	119
5	1	1	1	1	2	103
6	1	1	1	1	2	108
7	1	1	1	1	2	114
8	0	0	0	0	2	110
9	1	1	1	1	2	112
10	0	0	1	0	0	97
Total	8	7	8	7		
Girls						
1	0	1	1	1	2	125
2	0	0	0	1	2	121
3	1	1	1	1	2	110
4	0	0	0	0	2	109
5	0	0	0	0	2	119
6	1	1	1	1	2	122
7	1	1	0	1	2	116
8	1	0	1	1	2	149
9	0	1	0	0	2	93
10	1	1	1	1	2	103
Total	5	6	5	7		
			School 3			
Boys						
1	0	0	0	0	2	106
2	0	0	0	0	0	81
3	1	1	1	1	2	113
4	0	0	0	0	2	101
5	1	0	0	0	2	110
6	0	0	0	0	2	110
7	1	1	1	1	2	101
8	0	0	0	0	2	78
9	1	0	0	1	2	77
10	1	0	1	0	2	102
Total	5	2	3	3		
Girls						
1	1	1	1	1	2	116
2	0	0	0	0	2	93
3	0	0	0	0	1	95
4	1	0	0	0	2	119
5	0	0	0	1	2	110
6	1	1	1	0	2	96
7	0	1	0	0	2	108
8	0	0	1	1	2	101
9	1	1	1	1	2	97
10	1	1	1	1	2	82
Total	5	5	5	5		

	Item				Paper and pencil test	IQ
	1	2	3	4		
			School 4			
Boys						
1	0	1	0	0	2	94
2	1	1	1	1	2	126
3	0	0	1	0	2	108
4	0	1	0	1	2	102
5	1	1	1	1	2	117
6	0	0	0	0	2	109
7	0	0	0	0	2	97
8	0	0	1	0	2	110
9	1	1	1	1	2	97
10	1	0	0	0	2	117
Total	4	5	5	4		
Girls						
1	0	0	0	0	2	100
2	0	0	0	0	2	91
3	1	1	1	0	2	102
4	1	1	1	0	2	121
5	0	0	1	0	2	123
6	0	0	0	0	2	125
7	0	1	0	1	2	123
8	0	1	1	1	2	141
9	1	1	1	1	1	125
10	0	1	0	1	2	131
Total	3	6	5	4		
			School 5			
Boys						
1	1	0	0	0	2	121
2	0	0	0	0	2	101
3	0	1	1	1	2	86
4	0	0	0	0	2	87
5	0	0	0	0	2	78
6	0	0	1	0	2	100
7	1	0	0	0	2	107
8	1	1	1	1	2	96
9	0	0	0	0	2	131
10	1	1	0	1	2	113
Total	4	3	3	3		
Girls						
1	0	0	0	0	2	90
2	0	0	0	0	2	90
3	0	0	1	0	2	98
4	1	1	1	1	2	91
5	1	1	1	1	1	121
6	1	1	1	1	2	88
7	1	0	0	0	2	101
8	1	0	0	0	2	91
9	1	0	0	0	1	96
10	1	1	1	1	2	112
Total	7	4	5	4		

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